Testing of Hypothesis

Fisher's t-test (Test for Difference in Means of Two Independent Populations When σ_1, σ_2 are Unknown and Equal)

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Example :

Two brands of fertilizer, A and B, were applied to different plots of one-acre each; other conditions remaining the same. The yields (in quintals) are given below:

Fertilizer A : 18, 24, 36, 50, 34, 44, 32, 46

Fertilizer B : 33, 20, 24, 30, 42, 28, 47

Examine the significance of the difference between the mean yields due to the application of the two fertilizers.

Solution:

We want to test

$$H_0: \mu_1 = \mu_2$$

$$\text{against } H_a: \mu_1 \neq \mu_2$$

at α level of significance.

The test statistic is

$$t = \frac{\left|\bar{x}_1 - \bar{x}_2\right|}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

where $\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i}$, $\bar{x}_2 = \frac{1}{n_2}\sum_{j=1}^{n_2} x_{2j}$

and $$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1-n_2-2}$$

$$s_p^2 = \frac{\sum_{i=1}^{n_1}(x_{1i} - \bar{x}_1)^2 + \sum_{j=1}^{n_2}\left(x_{2j} - \bar{x}_2\right)^2}{n_1 - n_2 - 2}$$

Table for calculation of mean and variance

Fertilizer A (x_{1i})	$(x_{1i} - \bar{x}_1)$	$(x_{1i} - \bar{x}_1)^2$	Fertilizer B (x_{2j})	$(x_{2j} - \bar{x}_2)$	$(x_{2j} - \bar{x}_2)^2$
18	−17.5	306.25	33	1	1
24	−11.5	132.25	20	−12	144
36	0.5	0.25	24	−8	64
50	14.5	210.25	30	−2	4
34	−1.5	2.25	42	10	100
44	8.5	72.25	28	−4	16
32	−3.5	12.25	47	15	225
46	10.5	110.25			
Total: 284	**0**	**846**	**Total: 224**	**0**	**554**

Here $n_1 = 8$, $n_2 = 7$

$$\text{where } \bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i}\,,\ \bar{x}_2 = \frac{1}{n_2}\sum_{j=1}^{n_2} x_{2j}$$

$$\bar{x}_1 = 284/8 = 35.5 \qquad \bar{x}_2 = 224/7 = 32.0$$

$$s_p^2 = \frac{\sum_{i=1}^{n_1}(x_{1i} - \bar{x}_1)^2 + \sum_{j=1}^{n_2}\left(x_{2j} - \bar{x}_2\right)^2}{n_1 - n_2 - 2}$$

$$S^2 = (846 + 554) / (8 + 7 - 2)$$

$$= 1400/13$$

$$= 107.69$$

$$S = \sqrt{107.69} = 10.38$$

Calculated value of test statistic

$$t = \frac{\left|\bar{x}_1 - \bar{x}_2\right|}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$t = \frac{|35.5 - 32.0|}{10.38\sqrt{\frac{1}{8} + \frac{1}{7}}}$$

$$= 3.5 / 5.371 = \mathbf{0.652}$$

Decision rule:

From the t-table for $df = n_1 + n_2 - 2 = 13$ and $\alpha = 0.05$, we have $t_{13,\,0.025} = 2.16$ (t_{tab}). Since t_{cal} (0.652) < t_{tab} (2.16), we do not reject H_0 at 5% level of significance and conclude that the difference between the mean yields of the two fertilizers is not statistically significant.

Do your self

Example 1: Rice Yield under Two Fertilizers

Farmer	Fertilizer A	Fertilizer B
1	5.2	4.8
2	5.5	4.9
3	5.4	5.0
4	5.1	4.7
5	5.3	4.8
6	5.6	5.1
7	5.4	4.9
8	5.5	5.0

Question
Does Fertilizer A produce a significantly higher yield than Fertilizer B?

Example 2: Milk Yield of Indigenous vs Crossbred Cows

Cow	Indigenous (L/day)	Crossbred (L/day)
1	8	14
2	9	15
3	10	16
4	8	15
5	9	17
6	10	16
7	9	15
8	8	14

Question
Compare average milk production.